**Chapter 5**

**Integration**

**5.3. The Fundamental Theorem of Calculus**

**Section Exercises**

145. Two mountain climbers start their climb at base camp, taking two different routes, one steeper than the other, and arrive at the peak at exactly the same time. Is it necessarily true that, at some point, both climbers increased in altitude at the same rate?

Answer: Yes. It is implied by the Mean Value Theorem for Integrals.

147. Set . Find and the average value of  over .

Answer:  average value of  over  is 

**In the following exercises, use the Fundamental Theorem of Calculus, Part 1, to find each derivative.**

149. 

Answer: 

151. 

Answer: 

153. 

Answer: 

155. 

Answer: 

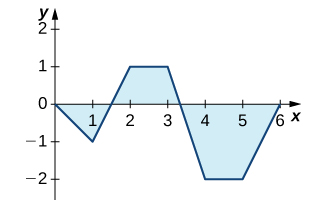
157. 

Answer: 

159. 

Answer: 

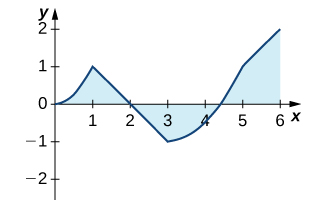
161. The graph of  where *f* is a piecewise constant function, is shown here.



* 1. Over which intervals is *f* positive? Over which intervals is it negative? Over which intervals, if any, is it equal to zero?
  2. What are the maximum and minimum values of *f*?
  3. What is the average value of *f*?

Answer: a. *f* is positive over  and , negative over  and , and zero over  and . b. The maximum value is 2 and the minimum is –3. c. The average value is 0.

163. The graph of  where *ℓ* is a piecewise linear function, is shown here.



* 1. Over which intervals is *ℓ* positive? Over which intervals is it negative? Over which, if any, is it zero?
  2. Over which intervals is *ℓ* increasing? Over which is it decreasing? Over which intervals, if any, is it constant?
  3. What is the average value of *ℓ*?

Answer: a. *ℓ* is positive over  and , and negative over . b. It is increasing over  and , and it is constant over  and . c. Its average value is .

**In the following exercises, use a calculator to estimate the area under the curve by computing *T*10, the average of the left- and right-endpoint Riemann sums using  rectangles. Then, using the Fundamental Theorem of Calculus, Part 2, determine the exact area.**

165. **[T]**  over 

Answer: , 

167. **[T]**  over 

Answer: 

169. **[T]**  over 

Answer:  

**In the following exercises, evaluate each definite integral using the Fundamental Theorem of Calculus, Part 2.**

171. 

Answer:  

173. 

Answer: , 

175. 

Answer: , 

177. 

Answer: , 

179. 

Answer: , 

181. 

Answer: , 

183. 

Answer: , 

185. 

Answer: , 

187. 

Answer: , 

189. 

Answer: , 

**In the following exercises, use the evaluation theorem to express the integral as a function**

191. 

Answer: 

193. 

Answer: 

**In the following exercises, identify the roots of the integrand to remove absolute values, then evaluate using the Fundamental Theorem of Calculus, Part 2.**

195. 

Answer: 

197. 

Answer: 

199. Suppose the rate of gasoline consumption in the United States can be modeled by a sinusoidal function of the form  gal/mo.

* 1. What is the average monthly consumption, and for which values of t is the rate at time t equal to the average rate?
  2. What is the number of gallons of gasoline consumed in the United States in a year?
  3. Write an integral that expresses the average monthly U.S. gas consumption during the part of the year between the beginning of April () and the end of September ().

Answer: a. The average is  since  has period 12 and integral 0 over any period. Consumption is equal to the average when , when , and when . b. Total consumption is the average rate times duration:  c. 

201. Explain why, if *f* is continuous over  and is not equal to a constant, there is at least one point  such that  and at least one point  such that .

Answer: If *f* is not constant, then its average is strictly smaller than the maximum and larger than the minimum, which are attained over  by the extreme value theorem.

203. A point on an ellipse with major axis length 2*a* and minor axis length 2*b* has the coordinates , .

* 1. Show that the distance from this point to the focus at  is , where .
  2. Use these coordinates to show that the average distance  from a point on the ellipse to the focus at , with respect to angle *θ*, is *a*.

Answer: a.; b. 

205. The force of gravitational attraction between the Sun and a planet is , where *m* is the mass of the planet, *M* is the mass of the Sun, *G* is a universal constant, and  is the distance between the Sun and the planet when the planet is at an angle *θ* with the major axis of its orbit. Assuming that *M*, *m*, and the ellipse parameters *a* and *b* (half-lengths of the major and minor axes) are given, set up—but do not evaluate—an integral that expresses in terms of  the average gravitational force between the Sun and the planet.

Answer: Mean gravitational force = .

**Student Project**

**A Parachutist in Free Fall**

1. How long after she exits the aircraft does Julie reach terminal velocity?

Answer: We want to find the time when Julie’s velocity reaches 176 ft/s. Solve



Julie reaches terminal velocity after 5.5 seconds.

3. If Julie pulls her ripcord at an altitude of 3000 ft, how long does she spend in a free fall?

Answer: We want to find out how much time has elapsed when Julie pulls her ripcord. Let *tp* denote this time. We know she pulls her ripcord after she has fallen 9,500 feet (from 12,500 feet to 3,000 feet), and we know we can figure out how far she has fallen by integrating her velocity function. So, we need to solve the following equation for *tp*.



Proceeding with the calculations, we get



Julie is in free fall for 56.727 seconds—nearly a minute!

5. How long does it take Julie to reach terminal velocity in this case?

Answer: Julie reaches terminal velocity after  seconds.

7. If Julie dons a wingsuit before her third jump of the day, and she pulls her ripcord at an altitude of 3000 ft, how long does she get to spend gliding around in the air?

Answer: In this case, Julie reaches terminal velocity in just  seconds. If we again use *tp* to denote the time when Julie pulls her ripcord, we want to solve



With a wingsuit on, Julie spends 216.597 seconds in freefall.

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